Final review

Math 485

December 13, 2013

1 Things to bring:

A calculator, a sheet of note (A4 - 2 sided).

2 Topics on the final:

Disclaimer: The final exam problems include, but are not limited to, the following topics:

2.1 Given problem:

Compute and show that the price of the Cash or nothing derivative satisfies the Black-Scholes PDE.

2.2 Topics covered in Midterm 1:

1. Probability: Conditional expectations (See Homework 1, also midterm 1 problem 1 a, c). Various random distributions and their moments: the uniform, the exponential, the normal, the binomial are the important ones. Central limit theorem (See Homework 1, midterm 1 problem 1b).
2. The one period model: Risk neutral measure, replicating portfolio, pricing of ANY derivative in 1 period model. What happens when we change certain elements of the one period model? (What if the stock has 3 outcomes? What if the stock pays dividend (see midterm 1 problem 7)). Can you find the replicating portfolio for any derivative in 1 period model?

3. The multiperiod model: Risk neutral measure, pricing of Euro-style derivatives: Euro put, call, pricing of American options (what is the relationship between American option and Euro option?), exotic options: Asian, Look-back, Barrier, Bermudan (See midterm 1 problem 5).
4. Conditional expectation in discrete time: How to compute certain conditional expectations: $E((S_7)^2|S_4)$? See homework 3 additional problems.
5. Martingale in discrete time: How to check some process is a martingale, sub-martingale, super-martingale in discrete time?

2.3 Topics coverd in Midterm 2

1. The continuous time Black-Scholes model. Brownian motion: properties of Brownian motion, its distribution. How to use the independent increment property to compute expression like $E(B_sB_t)$? Computation of other basic expressions involving Brownian motion: $E(B_t^2), E(e^{B_t})$.
2. Three Ito's formulas: how are they related? What is σ in the 2nd and 3rd Ito's formula? How to find σ? (See class notes on the lecture after the 2nd midterm).
3. Black-Scholes formula. Can you quickly change Black-Scholes formula to find the price of Euro-Call at time $t > 0$? Variations of Black-Scholes formula: Find $E[(S_T^2 - K)^+]$?
4. Black-Scholes PDE. How to verify the price of certain derivatives satisfy the Black-Scholes PDE?
5. Pricing of other Euro-style derivatives: $V_T = S_T^k, V_T = (S_T - K)^2$.

2.4 Topics covered after Midterm 2

1. Using Ito's formula to compute certain moments of Brownian motion: $E(B_t^4), E(B_t^6), E(e^{B_t})$. How about $E(\cos(B_t))$?.
2. Conditional expectation in continuous time: $E(B_t^2|B_s), E(e^{B_t}|B_s), E((B_t - K)^2|B_s)$? Compare / contrast the result with just simple expectation (the two formulas should share certain similarities). Direct application to finding the price of a Euro-style derivative at time t. Direct application to verifiying the price of a Euro-style derivative satisfying a Black-Scholes PDE.
3. Binomial approximation to continuous time model (with 5 time steps). How to compare the price given by Black-Scholes formula with the price given by the

Binomial model. In the lecture we use $r = 0$ for simplicity. What happens if $r \neq 0$? (How does this affect the choice of the X_k in the Binomial model? Does this affect what the risk neutral measure is in the Binomial model?) Application: Use the Binomial approximation to find price of American option in Black-Scholes model.
4. The Greeks: Δ, Γ, Θ. What are the Greeks used for? Find the Greeks of various Euro-style derivatives.
5. Some computations to think about: $E(|B_t|), E(e^{|B_t|}), E(e^{B_t^2})$.
6. Compute the explicit formula for some Ito's integral, for example $\int_0^t B_s^2 dB_s$ and check that it is a martingale.